Further Thoughts on 'Perfect' Labyrinths & How to Create Them

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I read with interest the articles in *Caerdroia* 35 by Andreas Frei ("The Cascading Serpentine")[1] and Tristan Smith ("A Daedalus for the 20th Century"),[2] along with information on Hebert[3] and Smith's[4] respective websites and Pierre Rosenstiehl's article "How the Path to Jerusalem at Chartres Separates the Birds from the Fishes."[5] The "cascading serpentine" path so aptly explained and exhibited in Frei's piece is clearly related to the "stretched H-array" mentioned by Rosenstiehl as essential for generating a labyrinth with alternating turns and straight runs - figure 1a shows Frei's graph of the Chartres thread side by side with Rosenstiehl's 'anamorphosis,' figure 1b. Both display the "stretched H-array": turn them sideways to see the outline of a series of diagonally connected "H's."). The top horizontal line in Rosenstiehl's figure is actually the labyrinth centre, not a circuit, so it can be ignored.

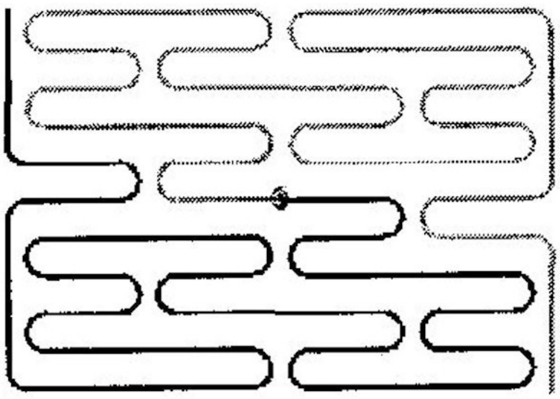


Figure 1a, above: Frei's graph for the Chartres labyrinth.

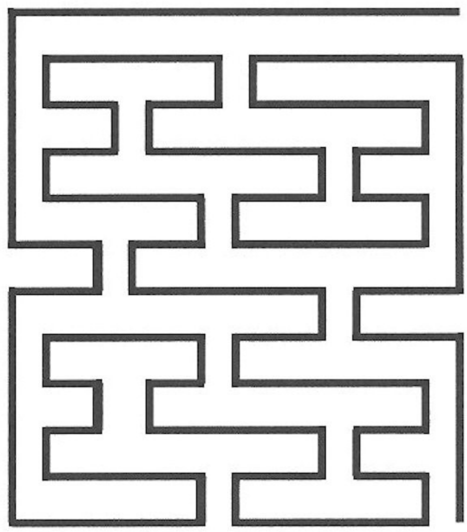


Figure 1b, right: Rosensteihl's 'anamorphosis' (unrolling) of the Chartres labyrinth

It is actually the thread of the Chartres labyrinth that divides the area of the labyrinth into two equal areas of the same shape (they are 'congruent'). That is, there is a white H-array, and a black H-array, interlocking, and the thread is the border between them! Here is an example (figure 2) of a stretched H-array (not Chartres). The edges that connect the levels (the "frame") have been cut off.

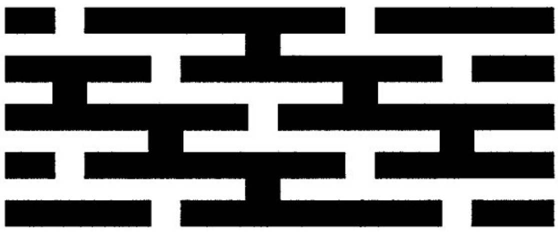


Figure. 2: Stretched H-array (no frame)

This same H-array property is also exhibited by just six of the twenty labyrinths discovered/generated by Hebert and Smith, specifically those with three repetitions of a "round course": Daedalus's labyrinths #9, 10, 11, 12, 15, and 17,[6] of which number 17 is Chartres itself. The other labyrinths have only one round course plus two folded motifs.

Perfect vs. Canonical

Rosenstiehl speaks of Chartres as the only 'perfect' medieval labyrinth of depth 12, i.e. 11 circuits with the centre being termed the 12^{th} level. His criteria include both a stretch H-array as well as a throat pattern consisting entirely of nested turns. Hebert prefers a less stringent definition and posits the 20 'canonical' labyrinths[7] found by himself and Smith which have reversible paths, a symmetrical/inverted throat pattern, and 3 repetitions total from the two types of 'motifs': round course or folded version (see figure 3). But the folded 'motifs' do not create a cascading serpentine path.

Figure 3:
(left) round course
(right) folded 'motifs'

Andreas Frei greatly simplifies the work in designing a labyrinth by using a 'cascading serpentine' path on a rectangular graph, which can then be transferred to a circular version with the 'goal' in the centre. This method has the advantage that it can be generalized to labyrinths of any size, meaning greater or fewer numbers of circuits (levels) or semi-axes (arms). Since the cascading serpentine corresponds to the stretched H-array, the only difference between Frei's definition and Rosenstiehl's is the throat pattern, which shows up on the rectangular graph as the extreme left-hand and right-hand connections between levels. This is the "frame" which Rosenstiehl "cuts off" to end up with the H-array (see figure 2).

Creating new labyrinth designs

How does knowing all this help us to create new labyrinth designs? Drawing on the ideas of both Hebert and Frei, we can do the following:

1) Choose a motif, which will determine minimum depth and # of arms. For example, a 5-step motif going forward 3 units (arms), back 2, forward 3, back 2 and forward 3. The minimum depth would be 5, the number of arms 5 (add: 3 – 2 + 3 – 2 + 3 = 5). In Frei's terms, this would be a 3-2 pathway sequence.

2) Repeat the motif an odd number of times (say, 3) to set up a cascading serpentine path. That will create a labyrinth of 15 circuits (depth 16 when the centre is included) and 5 arms.

3) Fill in the open areas of the Frei-style graph to make a complete path. Make sure to do this symmetrically, or the path will not be reversible!

4) Make symmetrical connections between the motifs to complete a reversible path. To be symmetrical, opposite corners and sides would be inversions of each other. Tony Phillips' website[8] also helps us here by noting that odd and even levels alternate in the level sequence. The path must exit on the opposite side from the entrance to be symmetrically reversible, and similarly, the first half of the thread will be the inversion of the second half. Figure 4 shows the result.

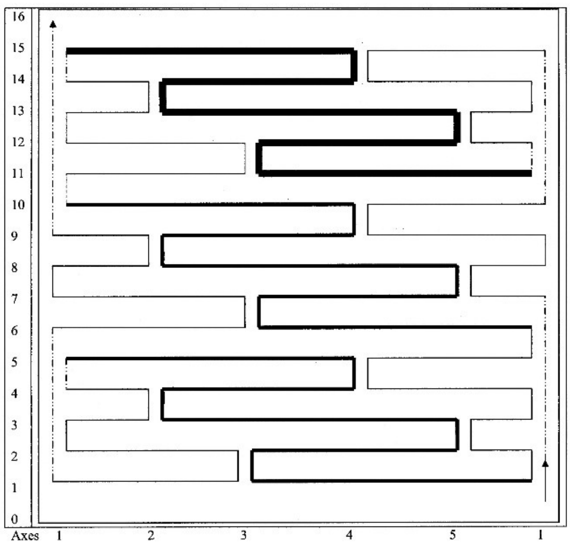
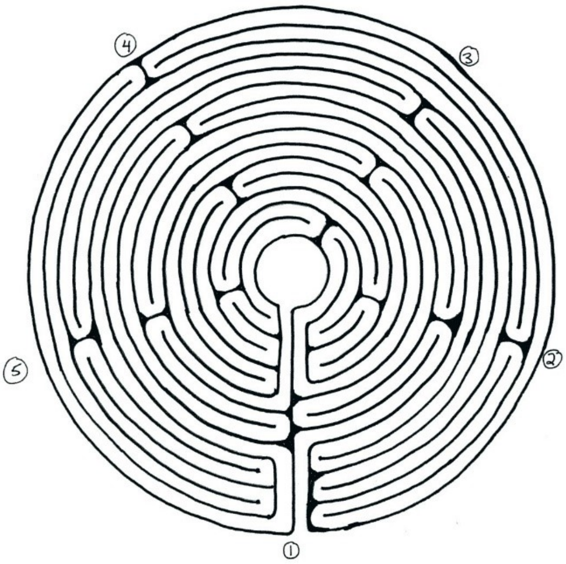


Figure 4: Frei-style graph for a 15-circuit, 5-arm labyrinth using a cascading serpentine pattern

The bold line shows the original 3-2+3-2+3 round course. The medium bold lines show the repetitions of the round course. The regular lines show the rest of the circuits filled in, and the dotted -.-.- lines show the connections made between levels to complete the path (the frame). Each side of the throat contains a 6-nest and an 8-nest (right & left sides of Axis 1). Axis = Arm.

Figure 5: The resulting 15-circuit, 5-arm labyrinth



Classification/Ranking of Labyrinths

There is still work to be done on comparing the labyrinths, similarities and differences: All of the six labyrinths listed above have the same pattern on the top semi-axis (arm). Using S to indicate Straight run (Single level) and T to indicate Turn (Two levels connected by turn) the top axes are all T S T S T S T. In addition, all of them have the same or inverted pattern on the left and right semi-axes (using Daedalus numbering, and reading across the labyrinth from left to right):

Labyrinths 9, 15 & 17: S T S T S T S S Centre S T S T S T S S

Labyrinths 10, 11 & 12: S S T S T S T S Centre S S T S T S T S

Thus, they differ mainly in the throat pattern, see figure 6. They can be further ranked according to how internally symmetrical (inside to outside) each side of the throat pattern is, from inside to out. For purposes of nested levels and turns, the entrance and centre are counted as levels 0 and 12.

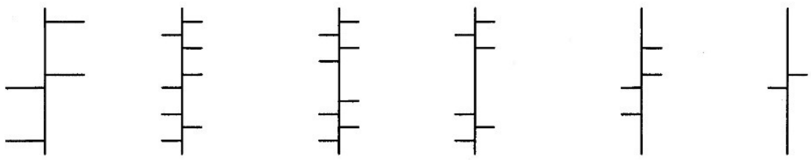
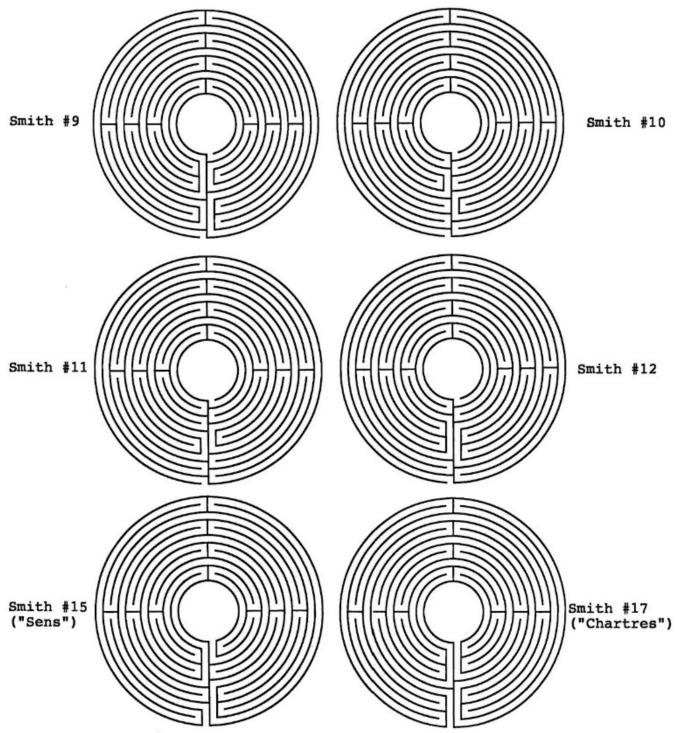


Figure 6 (above): Throat patterns (walls, not thread)

Figure 7: The six 'canonical' labyrinths that incorporate 3 round courses



Comparison of the six labyrinths that incorporate 3 round courses:

- #9: Each side holds a 6-nest and a 4-nest.
- #10: Each side has a 4-nest.
- #11: Each side has a 4-nest, like #10, but close to the centre of the throat (more symmetrical).
- #12: Each side has one 6-nest.
- #15: Sens: Each side has a 6-nest and a 4-nest, but the 4-nest is weaker, includes the outside & centre.
- #17: Chartres: each side has two 6-nests!

Chartres (#17) has essentially a cross within the throat, dividing it into four sets of nested turns (each set is what I will call a ‘6-nest’). Only #11 has comparable symmetry, but with a 4-nest (4 levels/2 turns nested) halfway into the centre, surrounded by two non-nested turns on each side (inward side and outward side). None of the other four have this internal symmetry within a side, however, 9 and 15 both have a nest in each quadrant of the throat, but it's a 4-nest balanced against a 6-nest. In #’s 10 and 12, each side has only one nest, not centred, though 12 comes closest.

In Conclusion

So, since there is currently no accepted definition as to what constitutes a ‘perfect’ or canonical labyrinth, I suggest it might make sense to rank them on degrees of symmetry: 1 degree for left and right axes being identical or reversed left to right; 1 degree for having left and right throat being identical or inversions of each other, and 1 degree for internal throat symmetry, inside to outside. By this reckoning, labyrinths #11 and #17 would have 3 degrees, labyrinths #9, 10, 12 and 15 would have 2 degrees, and the others (#’s 1-8, 13, 14, 16, 18-20) only 1 degree of symmetry.

I hope these thoughts will be a useful tool for thinking further about these medieval labyrinths!

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(Revised, October 2018)

Notes & References:

1. Frei, Andreas. “The Cascading Serpentine” *Caerdroia* 35 (2005), pp.19-26.
2. Smith, Tristan. “A Daedalus for the 20th Century” *Caerdroia* 35 (2005), pp.27-33, with forward by Jacques Hebert.
3. Jacques Hebert’s website: www.labyreims.com
4 Tristan Smith's website: www.otsys.com/~tsmith/labyrinths
5. Rosenstiehl, Pierre. “How the Path to Jerusalem at Chartres Separates the Birds from the Fishes” in *M.C. Escher: Art & Science*, Proceedings of the International Congress on M.C. Escher, Rome, Italy, 26-28 March, 1985. Elsevier, Holland.
6. All 20 are given at: www.otsys.com/~tsmith/labs.startWith1/pic.11.hebert.pdf
7. See: www.labyreims.com/e-annexe2.html
8. Tony Phillips’ website: www.math.sunysb.edu/~tony/mazes

Editor’s note:

The original printed version of this article contained an error in figure 7 (*Caerdroia 37*, p.48) that is corrected in this reprint edition.